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A NOTE ON OFFSHORE OIL FIELD DEVELOPMENT PROBLEMS AND SUGGESTED SOLUTIONS*

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This Note reviews some aspects of an approach to modeling the problem of developing offshore oilfields. An alternative method of solution is suggested and an associated problem, that of determining an optimal drilling schedule, is described.

As is sometimes true in management science work, simplifying assumptions lead to models which do not always coincide exactly with operational management problems. This Note, a comment on some recent work in the offshore oil field development area, indicates a possible discrepancy between one of management's real problems and some current models of that problem.

1. *The Problem*

We will not dwell on a description of the problem except to elaborate on Devine's [1] and Devine and Lesso's [2] discussion. A tentative map of the characteristics of an offshore oil field is made after the field is discovered and explored (using so-called step-out wells). An initial set of targets is decided upon based on this information.

Management faces the following problems:

a. Determination of location, size and number of platforms.
b. Targets drilled from each platform.
c. Targets drilled separately or as dual completions.
d. Drilling trajectories for each well (both single and dual completions).
e. Drilling schedule for each trajectory.

Decision e, the drilling schedule, refers to calculating the number of feet between successive trips (a trip is the action of pulling the drill sections up out of the ground, replacing the drill bit and replacing the drill assembly). The problem arises due to the tradeoff between the cost of time lost due to drilling with a worn bit and the cost of extra bits and tripping time. Problem d is a subject for current research.

Devine [1] and Devine and Lesso [2] have formulated a model to solve problems a, b and c. In that formulation, they treat the targets as fixed with the disclaimer:

> Because of the real world complexity, an exact analogue of the oil field development problem is not possible. Many factors add to this such as the stochastic nature of drilling costs, uncertainty about the exact nature of the field, current economic conditions, pollution standards, etc. [2]

This disclaimer assumes away an important aspect of the problem. A fixed set of well targets never exists as such since the wells are drilled sequentially; each well drilled in the field can add geological information which can possibly alter the position of future targets. To be operationally meaningful to management, the problem should be considered one of the sequential decision sort, with the information added by each well drilled potentially modifying (updating) the model. The development of such a sequential decision procedure could be a topic for future research.

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2. *Minimum Cost Drilling Schedule*

Devine and Lesso [2] mention that drilling costs are functions of horizontal distance from the platform to the target and of the depth of the target. This assumes that the method for drilling the hole is known. To test such an assumption as well as to get information about the drilling schedule itself, a study was made of drilling methods. After some analysis, it was established that two basic cost constants are needed to determine a drilling schedule through homogeneous material without changing drilling direction:

C_1 = cost per unit time associated with drilling and tripping.
C_2 = unit cost of a drill bit.

There are three components of the total cost to drill the well:

Drilling time cost:

$$C_1 \sum_{i=1}^{N} t(f_i) \tag{1}$$

where:

f_i = number of feet drilling with the ith bit,
N = number of bits used, and
$t(f_i)$ = rotating time to drill f_i feet.

t is normally a convex function of f_i : as a bit wears out, t increases at a greater and greater rate for Δf fixed.

Tripping time cost:

$$C_1\alpha \sum_{i=1}^{N} (N - i + 1)f_i \tag{2}$$

where α = trip time constant in hours/foot. Equation (2) follows since the first f_1 feet of piping must be pulled up N times (once for each bit replacement) while the last f_N feet are pulled up only once.

Bit cost:

$$NC_2 \tag{3}$$

The total drilling cost is obtained by summing (1), (2) and (3):

$$\text{Total cost} = C_1 \sum_{i=1}^{N} [t(f_i) + \alpha(N - i + 1)f_i] + NC_2 . \tag{4}$$

The minimum cost drill schedule is found by determining N and $\{f_i\}$ to minimize (4) subject to the constraints:

$$\begin{aligned} &\textstyle\sum_{i=1}^{N} f_i = \text{well length}, \\ &N, \text{ integer}, \\ &f_i \geqq 0,\ i = 1, \cdots, N. \end{aligned} \tag{5}$$

An *ad hoc* procedure using a simple nonlinear optimization routine was developed, systematically varying N and then solving for $\{f_i\}$. Solutions were reached normally within 5 seconds on an IBM model 360-75 and a set of costs for a 60 target field was developed in less than 20 minutes after suitably pruning the possibility space (eliminating physically infeasible dual completions). Dual completion costs, requiring an adjustment in drilling angle, were calculated in segments using the same procedure.

To tie these results to Devine and Lesso's, it was established that (4) could be related to the length and angle of the hole in the following way:

$$\text{Cost} = K_0 + (K_1 + K_2 L + K_3 L^2)/\cos \theta$$

where

$$\text{(6)} \qquad \begin{aligned} L &= \text{total hole length,} \\ \theta &= \text{angle of trajectory away from vertical, and} \\ \{K_i\} &= \text{constants to be estimated.} \end{aligned}$$

A good correlation exists between costs estimated from (6) and established from a sample of 30 wells ($R^2 = 0.87$). This is consistent with results reported by Devine and Lesso, although their cost formulation was slightly different than (6).

3. *Alternate Formulation of Devine's Problem*

Several years ago as part of a larger study, the problem that Devine [1] discusses was attacked. The resulting model is included for comparison with Devine's procedure.

Assume that from solving (4) a set of costs have been established as follows:

C_{ij} = minimum cost of a drilling schedule including targets i and j, $j \geqq i$.

We are now concerned with selecting particular well trajectories to connect various targets so that the combined cost is a minimum and all targets are reached exactly once. Arrange the targets in order of increasing depth from the surface, $i = 1, \cdots, n$ (n = number of targets). Define

$$X_{ii} = \begin{cases} 1 & \text{if there is a single completion trajectory to target } i \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ij} = \begin{cases} 1 & \text{if a dual completion trajectory passes through } i \\ & \text{and terminates at } j. \text{ Defined only for } j > i \\ 0 & \text{otherwise.} \end{cases}$$

The problem is then

$$\text{(7)} \qquad \begin{aligned} &\text{minimize } Z = \textstyle\sum_{i=1}^{n} \sum_{j=i}^{n} C_{ij} X_{ij} \\ &\text{subject to } \textstyle\sum_{i=1}^{k} X_{ik} + \sum_{j=k+1}^{n} X_{kj} = 1, \quad k = 1, \cdots, n \\ &\qquad X_{ij} = 0, 1. \end{aligned}$$

(7) is an all 0–1 integer programming problem. It was originally solved by Pierce's [3] algorithm for integer cutting stock problems. The method was tested on a 60 target field sample and a solution was obtained in 1.1 minutes on an IBM model 7094.

4. *Conclusions and Future Work*

It is not the goal of this Note to criticize the work of Devine and Lesso which, it is felt, can be of considerable strategic value. They themselves mention [2] that "the greatest attribute of the model should be to provide the decision makers with a better understanding of how the total development cost depends upon the various parameters." Devine and Lesso's approach, then, is likely to be of more value as a planning tool for testing the sensitivity of field development costs to changes in input assumptions than as an operational procedure for field development.

To be useful, an operational model of field development should incorporate the information gathered from each additional well drilled even if that information has only a minor effect on future decisions.

References

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