MANAGEMENT SCIENCE
Vol. 19, No. 2, October, 1972
*Printed in U.S.A.*

# A SYSTEM OF PROMOTIONAL MODELS*

AMBAR G. RAO† ‡ AND GARY LILIEN§

A system of promotional-effect models developed from certain behavioral assumptions about consumer buying habits is described. The same basic model structure is shown to be applicable to several types of gasoline marketing promotions and to various nongasoline promotions as well. Parameter estimation procedures and methods for calculating the effect of simultaneous promotions are discussed. The models were developed to be used with a computerized MIS for market planning and sales forecasting and are validated using actual sales data. Included are insights into the customer buying process that were revealed during the parameter estimation and updating procedures.

## Introduction

Computerization of marketing information systems often leads to the retention of data that previously were not gathered or, at least, not stored. The value of such a data base without associated tools such as management science models to extract actionable information is questionable (see, for example, Ackoff [1]).

Such model building efforts have frequently accompanied the development of sales forecasting procedures which incorporate not only information gleaned from historic time series studies, but also the expected effects of planned corporate strategies and anticipated competitive actions. Here the model building goal is to estimate the incremental effect of various marketing activities singly, and jointly, on sales. A system of models that provides such estimates can be used for

1. Development of good marketing plans. Management can simulate and evaluate the effects of alternative strategies together with competitive counterstrategies and then select the "best" plan.
2. Development of supporting plans once the marketing plan has been structured. The estimates could for instance be used as input for production scheduling and distribution models.
3. Monitoring sales and profit performance after the marketing plan has been implemented so that deviations from plan can be detected and appropriate action taken.

This third function, that of monitoring actual results and comparing them with plan, also fulfills a vital model building requirement: it provides systematic feedback which can adjust model parameters and enrich the models themselves. In as complex an area as marketing, where the aggregated responses of millions of individuals to corporate strategies are measured, an adaptive modelling capability is essential for long-term use of a system.

The authors were fortunate enough to participate in the development of a system for forecasting the gasoline sales of a major oil company. This paper describes some of the efforts to model the effects of various kinds of promotions on sales. In particular, we consider two kinds of promotions which were used extensively by oil companies in the sixties—competitive games and mass credit card mailouts. These particular types of promotions have declined in importance due to recent legislative changes, but the model structures appear to be valid for most station-based promotions. In addition, we

* Received August 1971; revised December 1971.

† New York University.
‡ Work done while this author was at Mobil Oil Corporation.
§ Mobil Oil Corporation, New York, New York.

indicate how the models can be adapted to reflect the effect on sales of price changes.

Although these models were developed in the specific context of gasoline marketing, the approach appears to be applicable to products and services that are sold in franchised outlets, where each outlet carries the "products" made by a single corporation. Examples are banks, snack food chains, tire stores, etc.

## Description of the Promotions and Data

Many variants of the competitive game promotion were offered to customers during the sixties. In essence, however, they were very similar: A customer would obtain a game form with a number or other identification from the promoting brand. Periodically, "winners" of the game would be selected by a random drawing. Thus the game was very similar to a lottery, although there were legal differences. The hope was that customers would be drawn from competing stations that were not participating in game promotions and that, after the game, some of these customers would be retained as regular purchasers. Later on, the promotions took on a more defensive aspect—they were instituted to avoid a loss of customers rather than to acquire new ones.

In the case of mailouts, credit cards were sent to residents in certain neighborhoods which satisfied some demographic and socioeconomic criteria. Again, a shift of customers to the mailout brand was the objective.

The data that were maintained were monthly shipments to stations in various market areas. There is generally a lag of about one week between a shipment occurring and a sale, although more precise estimates could, in principle, be obtained for each market. Shipment data were highly seasonal and in many markets exhibited a pronounced trend. Preliminary analysis showed that the effects of a credit card mailout increased slowly, reaching a peak in about four months. This level was then maintained for a substantial length of time. The effects of games on sales were, of course, more immediate.

It should be noted that unlike promotions for consumer-packaged goods, it was not expected that customers would buy *more* than their normal quantity. Sales gains could be achieved only by acquiring new customers—that is, customers who normally purchased gasoline sold by other companies.

## Model Concepts

Initial efforts to explain sales changes due to promotions included a regression analysis approach with an assortment of lagged and transgenerated variables. Acceptable fits to data were obtained, but the models were difficult to interpret and had poor predictive qualities. Hence the approach was dropped, and we decided to develop models from behavioral assumptions about individual consumer buying habits.

In such circumstances, it is often advantageous to assume that the population is heterogeneous in terms of its behavior; models of individual response to marketing action can then be developed and aggregated to appropriately reflect such differences. Many authors have used this approach to develop insightful models of customer behavior in different situations (see, for example, [3], [4]). In our case, however, not much a priori knowledge existed about customer heterogeneity. Therefore, we proceeded with an "expected value" type of approach, ignoring differences among individual customers. Since we were mainly interested in estimating expected changes in market share, however, we did not feel this was a significant omission.

It was hypothesized that the incremental gain in sales due to a promotion depends on the following:

1. The probability that a randomly chosen customer does not normally buy the promoting brand or brands. If we assume that the promoting brand(s) have a joint share of market equal to $m$, the required probability is an increasing function of $(1 - m)$. Since the sales increment due to a gasoline promotion arises out of purchases by *new* customers, rather than by increased purchase quantities by existing customers, the probability defined above can be regarded as the POTENTIAL ($P$) of a promotion.

2. The probability that a randomly chosen customer who does not buy the promoting brand(s) normally takes advantage of the promotion. This probability depends on two factors: (a) Consider a customer who has to travel an additional distance $d$ to take advantage of a particular promotion. The more attractive the promotion is, the larger $d$ can be and still have the customer respond to it. This factor is termed the STRENGTH ($S$) of a promotion. (b) For a given promotion, the smaller $d$ is, the higher the likelihood that the customer will respond to it, since it will mean less of a detour from his normal driving patterns. As $m$ increases, the distance $d$ will decrease. This is because as $m$ increases, the number of stations operated by the promoting brand(s) increases and the likelihood of finding one of these stations within a given radius of a fixed point will also increase. Thus the probability of finding a station with a promotion within a given distance is an increasing function of $m$. We shall call this probability the REACH ($R$) of a promotion.

Then, the probability that a randomly chosen customer will respond to a promotion is given by $P \times R \times S$. Consequently, the expected gain per customer in the market from a promotion is given by $V^* = P \times R \times S \times g$, where $g$ is the average quantity purchased by a randomly chosen customer during the promotional period. If there are $C$ customers in the market and $Cg$ is denoted by $G$, the total gasoline sold in the market, then the incremental sales gain to the promoting brands is given by

$$V = P \times R \times S \times G. \tag{1}$$

### Mathematical Formulation for a Game Promotion

The next step is to assume mathematical forms for the factors $P$ and $R$. Assume, for simplicity, that $P = (1 - m)^\beta$, $R = m^\alpha$, and $S = K$ (a constant), then $V_G = KG(1 - m)^\beta m^\alpha$, where $V_G$ is the volume (sales) gain due to a game promotion.

If this is indeed a reasonable form of the model, then we can make some observations about the exponents $\alpha$ and $\beta$. In particular, we assert $0 < \alpha < 1$ and $\beta \simeq 1$.

1. $0 < \alpha < 1$. By the time a brand (or brands in the case of simultaneous promotions) has a 20-25% share of the market, almost everybody should be within reach of a station which is conducting a promotion. Thus $\partial R/\partial m$ should be positive when $m$ is close to zero, and should gradually decline to a value $\gamma$ close to zero but positive when $m$ is close to 1.

Therefore, $\alpha m^{\alpha-1} > 0$ at $m = 0+$ and $\alpha m^{\alpha-1} \to \gamma$ as $m \to 1$. Hence, $0 < \alpha < 1$.

2. $\beta \simeq 1$. In equation (1), $V$ was defined as the volume gained by the promoting brand(s). Assuming that $G$ is constant for a given market at a given time, the loss by the nonpromoting brands is also $V$, and could be allocated as an approximation proportional to the shares of market of the nonpromoting brands. Consider a nonpromoting brand with share $m_0$, and let $V_L$ be the loss to this brand. Then

$$\begin{aligned} V_L &= KGm^\alpha(1 - m)^\beta(m_0/1 - m) \\ &= KGm^\alpha m_0(1 - m)^{\beta-1}. \end{aligned} \tag{2}$$

The expected sales volume of the brand when there are no promotions in the market is $Gm_0$, so that the proportional loss is

$$P_L = V_L/m_0G = Km^\alpha(1 - m)^{\beta-1}. \tag{3}$$

It is reasonable here to assume that $\partial P_L/\partial m \to 0$ as $m \to 1$. In other words, if the number of nongame playing brands is small ($m \to 1$), the effect of an additional brand having a game on proportional loss is also small (loss rate levels out). Thus, when $m$ is near 1,

$$K[\alpha m^{\alpha-1}(1 - m)^{\beta-1} - (\beta - 1)m^\alpha(1 - m)^{\beta-2}] \simeq 0$$

which implies $\beta \simeq 1$. Thus, equation (1) can be rewritten as

$$V = KGm^\alpha(1 - m), \qquad 0 < \alpha < 1, \tag{4}$$

and

$$P_L = Km^\alpha. \tag{5}$$

We have assumed in the above, for sake of generality, that there are several brands simultaneously using similar promotions, and their total market is given by $m$. If one of the promoting brands has a share $m_0$, and the gain is allocated in proportion to shares, the proportional gain by a brand with share $m_0$ when a total share of $m$ is being promoted is

$$P_G = KGm^\alpha(1 - m)(m_0/m)(1/m_0G) = Km^{\alpha-1}(1 - m). \tag{6}$$

Note also that $\lim_{m\to1} P_L = K$, so $K$ can be interpreted as an upper bound of the proportional loss that a company can incur by not participating in the promotion. It will be a reasonably tight bound if $m_0$ is small.

### Price Changes

The structure developed above can be modified to consider price changes. In most gasoline markets, changes within the group of major brands and within the group of independent brands are small relative to between-group price changes, the changes which will be considered here. A market-usual price difference between majors and independents can be established for most markets from historical price data. The strength of the draw of the price difference should be related to the magnitude of the change in that difference. Let

$\Pi_M{}^*$ = prevailing or market-usual major price,
$\Pi_I{}^*$ = prevailing or market-usual independent price,
$\Pi_M(t)$ = actual major price at $t$,
$\Pi_I(t)$ = actual independent price at $t$.

Then let

$$\delta = (\Pi_M(t) - \Pi_I(t)) - (\Pi_M{}^* - \Pi_I{}^*),$$

where $\delta$ thus represents the change in price difference at time $t$ in a given market.

To use reasoning analogous to that for games, a price differential change can be viewed as a promotion staged by the independent brands against the majors. Thus, letting $m_I$ represent the market share of independent brands (as a group), we have:

Potential = $(1 - m_I)$,
Reach = $m_I{}^\alpha$,
Strength = $f(\delta)$,

where $f(\delta)$ is some unknown function of price change.

Then the volume gain to the independent brands (as a group), due to the price change only, is given by

$$V_I = Gf(\delta)m_I^{\alpha}(1 - m_I). \tag{7}$$

If $f(\delta)$ can be approximated, for small $\delta$, by $K_0\delta$, then (7) simplifies to

$$V_I = GK_0\delta m_I^{\alpha}(1 - m_I); \tag{8}$$

and the proportional loss in sales to a major brand with share $m_0$ due to the price change by the independents is $K_0\delta m_I^{\alpha}$. Note that this loss can be positive or negative depending on the sign of $\delta$, that is, whether the independents increase or decrease their price relative to the majors.

### Mathematical Formulation for Mailouts

Modelling the effect of a credit card mailout is more complex than the two cases considered previously. We can modify the concept of potential here, however, and obtain a formulation quite similar to the previous ones. The basic problem is that the potential of a particular customer (and also the strength of the promotion to him) depends on whether he already purchases gasoline on credit or not. Let $m_j$ be the market share of the $j$th brand in a given market, and suppose a proportion $c_j$ of its sales are credit sales. We assume that there are $N$ brands in the market, and the promoting brand is Brand 0. The probability that a randomly chosen customer buys Brand $j$ on a given purchase occasion and purchases it on credit is $m_jc_j$. Similarly, the probability that he buys Brand $j$ and makes a cash purchase is $m_j(1 - c_j)$. Thus the probability that a randomly chosen customer buys any brand but Brand 0 on credit is given by $\sum_{j=1}^{N} m_jc_j$, and the probability that the purchase is on cash is $\sum_{j=1}^{N} m_j(1 - c_j)$. Now let the strength of the promotion be $K_1$ for a customer of the first and $K_2$ for a customer of the second of the two types discussed above. Let $n$ be the number of cards mailed out. Then the volume gained by the promoting brand, recalling that the reach of the brand is $m_0$, is given by

$$V_c = m_0^{\alpha}f(n)G[K_1 \textstyle\sum_{j\neq 0} m_jc_j + K_2 \sum_{j\neq 0} m_j(1 - c_j)], \tag{9}$$

where $f(n)$ is some unknown function of $n$, the mailout size. If we assume that $f(n)$ is approximately a linear function of $n$ given by $n/p$, where $p$ is the car population in the market,

$$V_c = G(n/p)m_0^{\alpha}[K_1 \textstyle\sum_{j\neq 0} m_jc_j + K_2 \sum_{j\neq 0} m_j(1 - c_j)]; \tag{10}$$

and the proportional gain to the company is $P = V_c/m_0G$. The above development is for the case when Brand 0 is the only one conducting a mailout promotion. If another brand, say Brand $K$, has a simultaneous mailout of $L$ cards, then the gain to Brand 0 is

$$V'_c = V_c - G(L/p)m_K^{\alpha}[K_1m_0c_0 - K_2m_0(1 - c_0)],$$

where the second term on the right is the gain Brand $K$ would obtain from Brand 0.

### Interaction Effects

Thus far, we have indicated how game, credit card and pricing effects are modelled under the assumption that only one effect is present in a market at a time. If more than one promotion occurs, the joint effect of the two promotions would not, in general, be expected to be simply the sum of the individual effects. We therefore face the problem of modelling interaction effects.

Suppose there is both a game and a credit card promotion in a market at a certain time. It is then necessary to calculate

$$V_G \cup V_C = V_G + V_C - V_G \cap V_C,$$

where $V_G$ is game-promotion volume, $V_C$ is credit card promotion volume.

If we let $G$ = total market volume, then

$V_G/G$ = proportion of total market volume attracted to the game promotion,

$V_C/G$ = proportion of volume attracted to credit card promotion.

We now assume that these volumes will have a one-to-one relationship with respective customer sets. Let

$F$ = total population size,

$f_G$ = members of the population attracted by the game,

$f_C$ = members of the population attracted by the credit card promotion.

If either $f_C$ or $f_G$ is a proper subset of the other (all those who are attracted by a credit card promotion would have been attracted by a game, for example) then

$$f_C \cap f_G = \min \{f_C, f_G\}.$$

These populations are probably at least partially distinct, so this should overstate the interaction. On the other hand, if these populations can be considered independent random samples (with replacement) from $F$, then the proportion of the population we would expect to have duplicated is the product $(f_G/F) \cdot (f_C/F)$. Multiplying by $F$ to convert this proportion to an absolute population size implies:

$$f_G \cap f_C = \frac{f_G \cdot f_C}{F^2} \cdot F = \frac{f_G \cdot f_C}{F}.$$

This should understate the interaction, since these populations are hardly likely to be independent. Making the connection with the associated volumes, we can now state that

$$\min (V_G, V_C) > V_G \cap V_C > V_C \cdot V_C/G.$$

An intuitively appealing approach to modelling $V_G \cap V_C$ is to formulate this intersection as a convex combination of these bounds:[1]

$$(11) \qquad V_G \cap V_C = \lambda \min \{V_G, V_C\} + (1 - \lambda) V_G \cdot V_C/G, \qquad 0 < \lambda < 1.$$

An estimation procedure for $\lambda$ along with the other model parameters is developed in the next section.

### Estimation of Parameters

Consider a market where some promotion or combination of promotions is being conducted by a company.

Assume the following relationship:

$$(12) \qquad Y = \mu + \text{Proportional change in sales due to promotions} + \epsilon,$$

where

$Y$ = Actual sales/Forecast sales for the period,

$\mu = E(Y \mid \text{no Promotions})$,

$\epsilon$ = random error ($E(\epsilon_i) = 0$, $\{\epsilon_i\}$ uncorrelated over time).

Note that $\mu$ here represents 1 + proportional forecasting bias. Since our model gives us expressions for the second term on the right of (12), we can obtain estimates for

[1] The authors are indebted to John D. C. Little for this suggestion.

$\alpha$, $\lambda$, and the $\{K_i\}$ by a least squares procedure if we know $\mu$. To estimate $\mu$ we reason as follows:

Although the oil industry had promoted gasoline in the past using station-based promotions, these promotions never had as large an effect as games and credit card mailouts. Thus the effect of historical promotions on sales could be treated as a component of random error, and a time series analysis of historical sales was indirectly used to obtain estimates for $\mu$ in various markets.

Letting $Y_{it}$ refer to Actual Sales/Expected Sales in market $i$ at time $t$, we can measure the constants for the game effect as follows: Suppose a set of markets $\{I\}$ is isolated for which, at $t$, the only promotional disturbance is a game (i.e., prices are stable, no credit card mailout). Then,

$$Y_{it} = \mu_i + G_{it} + \epsilon_{it}, \qquad i \in \{I\},$$

where $G_{it}$, the effect of games (referring to (5) and (6)),

$= P_G$ if the brand has a game promotion in market $i$ at time $t$,

$= P_L$ if the brand does not have a game promotion in $i$ at time $t$.

TABLE 1

*Model-Estimated Changes in Market Share**

| Market Share in Games | Company Has Game? | Change in Price Diff. | Market Share of Independents | Part of Population in Mailout | Percent Change in Market Share |
|---|---|---|---|---|---|
| 0 | — | 2¢ | 0.30 | — | −12% |
| 0 | — | 1¢ | 0.20 | 0.05 | −2% |
| 0 | — | — | 0.10 | 0.10 | +8% |
| 0 | — | −1¢ | 0.10 | 0.15 | +17% |
| 0.15 | Y | 2¢ | 0.10 | — | +10% |
| 0.15 | Y | 2¢ | 0.20 | 0.05 | +11% |
| 0.15 | Y | 1¢ | 0.30 | 0.10 | +18% |
| 0.15 | Y | 1¢ | 0.10 | 0.15 | +21% |
| 0.15 | N | — | 0.20 | — | −3% |
| 0.15 | N | −1¢ | 0.30 | 0.05 | +6% |
| 0.15 | N | −1¢ | 0.10 | 0.10 | +9% |
| 0.15 | N | −2¢ | 0.20 | 0.15 | +19% |
| 0.25 | Y | 2¢ | 0.30 | — | — |
| 0.25 | Y | 2¢ | 0.10 | 0.05 | +4% |
| 0.25 | Y | 1¢ | 0.20 | 0.10 | +10% |
| 0.25 | Y | 1¢ | 0.30 | 0.15 | +12% |
| 0.25 | N | — | 0.10 | — | −4% |
| 0.25 | N | −1¢ | 0.20 | 0.05 | +5% |
| 0.25 | N | −1¢ | 0.30 | 0.10 | +10% |
| 0.25 | N | −2¢ | 0.10 | 0.15 | +17% |
| 0.35 | Y | −2¢ | 0.20 | — | +18% |
| 0.35 | Y | −2¢ | 0.30 | 0.05 | +21% |
| 0.35 | Y | −1¢ | 0.10 | 0.10 | +16% |
| 0.35 | Y | −1¢ | 0.20 | 0.15 | +21% |
| 0.45 | N | — | 0.30 | — | −5% |
| 0.45 | N | 1¢ | 0.10 | 0.05 | −5% |
| 0.45 | N | 2¢ | 0.30 | — | −16% |

* Assumptions: 50% of market buys using cash.
The company has an 8% expected market share.
No other company has a credit card mailout.

Let $I = \{I_G, I_{NG}\}$ where $\{I_G\}$ is that subset of the markets where the company has a game, $\{I_{NG}\}$ where the company does not. Referring to equations (5) and (6) again:

$$Y_{it} = \mu_i + Km_{it}^{\alpha-1}(1 - m_{it}) + \epsilon_{it}, \qquad i \in \{I_G\}, \tag{13a}$$

$$Y_{it} = \mu_i - Km_{it}^{\alpha} + \epsilon_{it}, \qquad i \in \{I_{NG}\}. \tag{13b}$$

Least squares estimates of $K$ and $\alpha$ can now be made from (13) using a nonlinear optimization technique. Estimation procedures for the other parameters are similar.

It should be noted that our goal at this stage of the procedure is to obtain working parameter estimates. Since we derived no closed-form results for the estimates, we can make no claim for their statistical properties. However, we do review their predictive qualities in the next section.

## Model Characteristics and Results

Once parameters had been obtained, the model system was incorporated into the existing time-series forecasting package and was used to predict sales in various markets where promotions were being conducted. Figure 1 compares, in a market that was used for validation, the results of the time series forecasts alone (the alternative system and the one in use at the time) with the results after the addition of the promotions models. The lag in pickup of the July–August 1967 sales spurt was attributed to an advertising campaign preceding the mailout, and advertising promotions were not treated in the model structure.

In a test of 19 markets, the variance of the error between twelve monthly actual and predicted sales figures was reduced by over 50%.

Table 1 shows a sample of the output of the models as estimated in June 1968 for a variety of situations. For example, the fifth line indicates that when 15% of the market, including the company of interest, is participating in a game promotion, when the market usual price difference has been increased by 2¢ and independents have 10% of the market (with no credit card mailout in effect), a 10% sales boost can be expected.

Some interesting insights into consumer behavior were revealed from updating parameter estimates. The game-strength parameter, $K$, was approximately half its original value one year after games were introduced. The strength of a credit card mailout was found to peak about four months after the original mailing and then slowly decline. Its effective 'half-life' (time until half the peak incremental sales were pro-

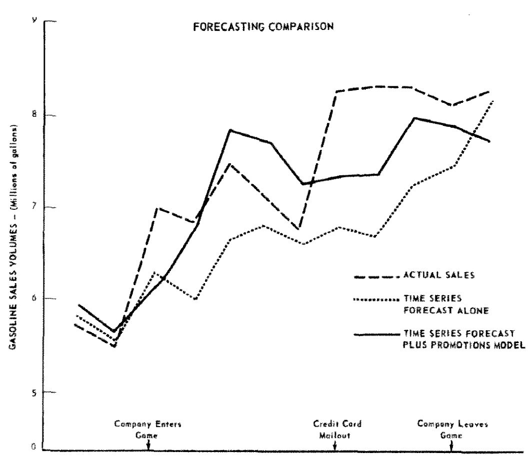


FIGURE 1

duced) was about one and a half years. Price parameters showed the largest sample-to-sample variation indicating either highly erratic consumer price consciousness or possible weakness in the price-model structure.

## Implementation

In addition to its use in a sales forecasting procedure, this system could be used in several ways: With the addition of competitive information and competitive response assumptions, optimal allocations of promotional expenses within and across markets could be developed. Alternately, the system could be used more as an "aid-to-judgment" to simulate the effects of promotional strategies in different competitive situations.

Because of the amount of additional competitive data and model-building effort needed for promotional expense allocations, the "aid-to-judgment" approach was chosen initially. An on-line interactive program was developed to make the system of models accessible to marketing management and to increase the probability that they be properly used for planning and control. A manager could access the models through a console and experiment with promotional assumptions and timings. Even more importantly, the effects of assumed or anticipated competitive actions and reactions could be examined. Little [2] discusses this method of using models.

Due to the termination of the specific promotions discussed in this paper, we could not complete the cycle of development-experimentation-model refinement as we had planned at the beginning of the project. However, a framework for so doing had been developed and could be used in the future for other station-based promotions.

### References


1. ACKOFF, R. L., "Management Misinformation Systems," *Management Science*, Vol. 14, No. 4 (December 1967), pp. B147–156.
2. LITTLE, J. D. C., "Models and Managers: The Concept of a Decision Calculus," *Management Science*, Vol. 16, No. 8 (April 1970), pp. B466–485.
3. MASSY, W. F., MONTGOMERY, D. B. AND MORRISON, D. G., *Stochastic Models of Buying Behavior*, The MIT Press, Cambridge, Mass., 1970.
4. RAO, A. G., *Quantitative Theories in Advertising*, John Wiley & Sons, New York, 1970.